Nice work!

Marjorie + I worked together on all of these! Good!

Supercorrection Four-Point Form Name

1j Convince me that you now understand the concept- connect to previous HW, notes, etc. Be sure to also explain the error(s) that you made.

Original 0 /4 Supercorrection 4 /4

I honestly have no idea why I left this one blank on the test. I think I panicked: I glanced at it quickly and when I didn't know the answer right away, I just skipped. Had I paused and taken 2 more seconds to look at it, I would have discovered that the answer is quite obvious if you know those "7 important Log properties" which I do. I think I have good proof/explanation below.

Correct solution:

j) $\log_a x^y = (\log_a x)^y$ False

Nice!

b/c $\log_a x^y = y \cdot \log_a x$ NOT $(\log_a x)^y$

$a=2$
$x=4$
$y=3$

$\log_2 4^3 =$
$4^3 = 64$
$\log_2 64 = \boxed{6}$

or

$\log_2 4^3 =$
$3 \cdot \log_2 4 =$
$3 \cdot 2 = \boxed{6}$

BUT

$(\log_2 4)^3 = 2^3 = \boxed{8}$

$\boxed{6 \neq 8}$

2 Convince me that you now understand the concept- connect to previous HW, notes, etc. Be sure to also explain the error(s) that you made.

Original 3.9 /4 Supercorrection 4 /4

I'm not sure if you marked this one right or wrong. I know I got the correct answer, but I wrote it in both fraction form and decimal form. You crossed out the decimal. (The decimal isn't the smallest value, because you can always go out to another decimal place.

Good!

Correct solution: smallest value of x for which $3^x \geq 1{,}000{,}000$

$$x = \frac{\log 1{,}000{,}000}{\log 3} \quad \text{or} \quad \frac{\ln 1{,}000{,}000}{\ln 3}$$

Supercorrection Four-Point Form Name: [redacted]

#3 Convince me that you now understand the concept- connect to previous HW, notes, etc. Be sure to also explain the error(s) that you made.
Original 3 /4 Supercorrection 4 /4

I knew what I had to do for this problem, but I still made a mistake. I knew I had to use the given information to find the # of bacteria after 12 hours, but when writing the equation, I mistakenly did $900 = 400e^{k(0)}$ instead of $900 = 400e^{k(4)}$. I know you have to use the 2nd equation to find k because there are 900 bacteria after 4 hours, not after 0 hours.

Correct solution:

$B = B_0 e^{kt}$

$t=0, B=400$

$t=4, B=900$

$900 = 400e^{k(4)}$

$\frac{900}{400} = \frac{\cancel{400}e^{4k}}{\cancel{400}}$

$\frac{900}{400} = e^{4k}$

$\frac{\ln \frac{900}{400}}{4} = \frac{4k}{4}$

$\boxed{k = .2027}$

$B = 400e^{(.2027)(12)}$

$B = 400e^{2.4324}$

$\boxed{B = 4554.5}$ ← Can you have a fraction of a bacterium?

There will be 4,554.5 bacteria after 12 hours, not 1,900.

#5d. Convince me that you now understand the concept- connect to previous HW, notes, etc. Be sure to also explain the error(s) that you made.
Original 2 /4 Supercorrection 4 /4

On the test, I wrote the inverse of $y = 2^x$ (which is the log... $\log_2 y = x$), but I didn't write it in y= form. I knew I didn't have the answer exactly correct, and the correct answer was right on the tip of my... brain, but it just didn't come. It's so obvious to me now that you swap x's and y's and go from an exponential to a log to get the inverse.

Correct solution:

Write the equation of the inverse of $y = 2^x$ in y= form.

$y = 2^x$

$\log_2 y = x$

$\log_2 x = y$

$\boxed{y = \log_2 x}$

or

$y = 2^x$

$x = 2^y$

$\log_2 x = y$

$\boxed{y = \log_2 x}$

Supercorrection Four-Point Form Name:

6 Convince me that you now understand the concept- connect to previous HW, notes, etc. Be sure to also explain the error(s) that you made.

Original 2 /4 Supercorrection 4 /4

I gave myself a 2 on this problem b/c when I encountered it on the test, I knew I was going to have to use those "7 Important Log Properties" to solve it... I just didn't use them in the right way. I saw that the division of x^2 could be written as a subtraction of $\log x^2$, but I missed that $\log 100\sqrt{y}$ could be written as the addition of 2 logs: $\log 100 + \log\sqrt{y}$.

Correct solution:

$$\log\frac{100\sqrt{y}}{x^2} = \log 100 + \log\sqrt{y} - \log x^2$$

$$2 + \tfrac{1}{2}\log y - 2\log x$$

$*\ \log\sqrt{y} = \log y^{1/2} = \tfrac{1}{2}\log y$

$*\ \log x^2 = 2\log x$

b/c of the amazing 7 important Log Properties ☺

8 Convince me that you now understand the concept- connect to previous HW, notes, etc. Be sure to also explain the error(s) that you made.

Original 1 /4 Supercorrection 4 /4

Marjorie and I pondered this one together. When making a sad attempt at this problem on the test, I knew that the range of $f(x)$ is equal to the domain of $f^{-1}(x)$... and that the domain of $f(x)$ is equal to the range of $f^{-1}(x)$. I knew I was going to have to use that somehow, but I had no clue how to get started. thurough explanation below. ↓

Correct solution:

$f(x) = \frac{e^x+7}{2}$ find range of $f(x)$ using $f^{-1}(x)$

$y = \frac{e^x+7}{2}$

$x = \frac{e^y+7}{2}$

$2x = e^y + 7$

$2x - 7 = e^y$

→ $\ln 2x - 7 = y$

$f^{-1}(x) = \ln(2x-7)$

The domain of this ($f^{-1}(x)$) is equal to the range of $f(x)$, so, I have to find the domain of this.

→ You can't take the natural log of zero or a neg. #, so...

$2x - 7 > 0$

$+7 \quad +7$

$2x > 7$

$x > \frac{7}{2}$

→ domain of $f^{-1}(x)$ = range of $f(x)$

domain $f^{-1}(x)$: $x > \frac{7}{2}$

Therefore,

Range $f(x)$: $x > \frac{7}{2}$

Supercorrection Four-Point Form Name:

#__9__Convince me that you now understand the concept- connect to previous HW, notes, etc. Be sure to also explain the error(s) that you made.
Original __1__/4 Supercorrection__4__/4

I basically had no idea how to attempt this problem, I just made a not-so-educated guess. Stevie really helped me with this one by explaining that with the equation $\frac{1}{2}^x = 4$, x must be a negative number. The same is true for any fraction raised to a power: the power must be negative for it to be equal to a whole number. I suppose one of the exponent rules can be applied to this problem. $a^{-n} = \frac{1}{a^n} \Rightarrow \frac{1}{2}^{-2} = \frac{1}{\frac{1}{2}^2} = \boxed{4}$

Good!

Correct solution:

$a > b$

$\log_b a$ is less than 0

$a = ?$

$b = ?$

$b = \frac{1}{2}$

$a = 4$

$\log_{\frac{1}{2}} 4$

$\frac{1}{2}^x = 4$

* a fraction needs to be raised to a negative number to get a whole number. *

$(\frac{1}{2})^{-2} = 4$

$\log_b a = \boxed{-2}$ less than zero!!

$b = \frac{1}{4}$

$a = 16$

$\log_{\frac{1}{4}} 16$

$\frac{1}{4}^x = 16$

$(\frac{1}{4})^{-2} = 16$

$\log_b a = \boxed{-2}$

less than zero!!

#__10__Convince me that you now understand the concept- connect to previous HW, notes, etc. Be sure to also explain the error(s) that you made.
Original __2__/4 Supercorrection__4__/4

Ahhh! I would have had this one correct... but I copied the problem down wrong when I was doing it on the test! I did all the right steps, but I wrote $\log_a(x-4)$ instead of $\log_a(x+4)$... 1 negative sign!! So anyway, the problem's fairly easy: you just use the Log properties to combine the two $\log_a$'s, and after that it's plain algebra.

Correct solution:

solve for x: $\log_a x + \log_a(x-2) = \log_a(x+4)$

$$\frac{\cancel{\log}_a(x^2-2x)}{\cancel{\log}_a} = \frac{\cancel{\log}_a(x+4)}{\cancel{\log}_a}$$

$$x^2 - 2x = x + 4$$
$$-x-4 \quad -x-4$$

$$x^2 - 3x - 4 = 0$$

$a = 1 \quad b = -3 \quad c = -4$

$$\frac{3 \pm \sqrt{9+16}}{2} = \frac{3 \pm 5}{2} =$$

$\boxed{4}$ ~~or~~ $\boxed{\cancel{-1}}$ why not?

$\boxed{x = 4}$

Supercorrection Four-Point Form Name

\# 11 Convince me that you now understand the concept- connect to previous HW, notes, etc. Be sure to also explain the error(s) that you made.
Original 0 /4 Supercorrection 4 /4

I had no idea how to attempt this on the test, that's why I gave myself a zero.

a) we know from the exponent and radical rules that $\frac{1}{b^2} = b^{-2}$
we know from the Important Log Properties that $\log_a b^{-2} = -2\log_a b$.

b) we learned in the homework to take log of both parts of these expressions to solve them.

c) we know from the Important Log Properties that $\log_a ab = \log_a a + \log_a b$.

Correct solution:

$\log_a b \approx 2.3219$

a.) $\log_a \frac{1}{b^2} =$

$\log_a b^{-2} =$

$-2\log_a b$

2.3219
$\times \; -2$

$\boxed{-4.6438}$

b.) $\log_b a =$

$\frac{\log_a a}{\log_a b} = \boxed{\frac{1}{2.3219}}$

c.) $\log_a ab =$

$\log_a a + \log_a b =$

$1 + 2.3219 = \boxed{3.3219}$

Other solutions?

\# 12 Convince me that you now understand the concept- connect to previous HW, notes, etc. Be sure to also explain the error(s) that you made.
Original 3 /4 Supercorrection 4 /4

I gave myself a 3 on this one because I knew I had to use fractions to cancel out other fractions (and I had the $2^3 = 8$ part in there, also). I understand now that you multiply both sides by 4/3 and this cancels out the 3/4 attached to x. You then have $x = 8^{4/3}$. 8 can be written as 2^3, which makes it easy to see that $(2^3)^{4/3}$ equals 2^4... which equals 16.

True!

Correct solution:

Solve for x: $x^{\frac{3}{4}} = 8$

$(x^{\frac{3}{4}})^{\frac{4}{3}} = 8^{\frac{4}{3}}$

$x = 8^{\frac{4}{3}}$

$x = (2^{\frac{3}{1}})^{\frac{4}{3}}$

$x = 2^4$

$\boxed{x = 16}$

$\sqrt[\frac{3}{4}]{8} = 8^{\frac{1}{3/4}} = 8^{4/3}$

Supercorrection Four-Point Form Name

#13b Convince me that you now understand the concept- connect to previous HW, notes, etc. Be sure to also explain the error(s) that you made.
Original 2 /4 Supercorrection 4 /4

I don't know what I was thinking. I guess maybe I thought that you wouldn't put something this obvious on the test... so I freaked out and over-thought it... and decided that $h(g(x)) = 1^x$... not x. This is especially silly considering I got both part a and part c correct. And in part c I said that the functions have an inverse relationship (very true)...

Correct solution:

b) Find $h(g(x))$.

$7^{\log_7 x} = \boxed{x}$

so how could something as untrue as $h(g(x)) = 1^x$ work out as the inverse of $g(h(x))$, which $= \boxed{x}$?!! Silly me

#_____ Convince me that you now understand the concept- connect to previous HW, notes, etc. Be sure to also explain the error(s) that you made.
Original ____/4 Supercorrection____/4

Correct solution:

ALGEBRA 2 HON $\frac{84}{104}$ = 80% Much better! Name: [redacted]

A Finely Crafted O'Brien Unit 5 Test

Calculator Section: You may use a calculator. Show all work and circle your answer. Use your time wisely; you will be able to earn additional credit after the timed portion of the test by completing Supercorrections. When you finish, put away your calculator and you come up to get the non-calculator part- you may continue to work on both sections without your calculator.

1. In the following statements, $a > 0,\ a \neq 1$ and $x > 0,\ y > 0$. Determine whether each is true or false.

7

a. $\log_a a = 1$ True

b. $\log_a 1 = a$ False

c. $\log_a 0 = 1$ False

d. $\log_a 1 = 0$ True

e. $\log_a xy = \log_a x + \log_a y$ True

f. $\log_a (x+y) = \log_a x + \log_a y$ False

g. $\log_a (x+y) = \log_a x \cdot \log_a y$ False

h. $\log_a xy = \log_a x \cdot \log_a y$ False

i. $\log_a x^y = y \log_a x$ True

j. $\log_a x^y = (\log_a x)^y$ False

2. Find the smallest value of x for which $3^x \geq 1{,}000{,}000$.

$\log_3 1{,}000{,}000 = x$

12

$x = \dfrac{\log 1{,}000{,}000}{\log 3}$ $\approx$ ~~12.57541965~~

or $\dfrac{\ln 1{,}000{,}000}{\ln 3}$

3. The number of bacteria B in a culture increases according to the equation $B = B_0 e^{kt}$. There were 400 bacteria at time $t = 0$ and 900 bacteria at time $t = 4$ hours. Will there be 1900 bacteria after 12 hours? If not, how many will there be?

6

t	bacteria
0	400
4	900

$900 = 400e^{k(4)}$

$\frac{900}{400} = \frac{400e^{4k}}{400}$

$\ln \frac{900}{400} = \frac{4k}{4}$

4

$B = B_0 e^{kt}$

$900 = 400e^{k(0)}$

$\frac{900}{400} = \frac{400e^{k}}{400}$

$\ln 2.25 = \cancel{\ln e}^{k}$

$\boxed{K = 0.812}$

$B = 400e^{0.812(12)}$

$B = 400e^{9.744}$

$\boxed{B = 6820644.64 \text{ after 12 hours}}$ ✗

That many! Wow!

$\boxed{K = .2027}$

$B = 400e^{(.2027)(12)}$

$B = 400e^{2.4324}$

$\boxed{B = 4554.47}$

(NO)

4. At the right is a "solution" to the equation $100 = 18e^{4k}$.

$$100 = 18e^{4k}$$
$$\ln 100 = \ln\left(18e^{4k}\right)$$
$$\ln 100 = \ln 18 \ln\left(e^{4k}\right)$$
$$\frac{\ln 100}{\ln 18} = \ln\left(e^{4k}\right)$$
$$\frac{\ln 100}{\ln 18} = 4k$$
$$\frac{\ln 100}{4\ln 18} = k$$
$$k \approx 0.398$$

a. Check the answer $k \approx 0.398$ back in the equation $100 = 18e^{4k}$ and show that it doesn't work.

$100 = 18e^{4(0.398)}$ $\boxed{100 \neq 88.44}$

$100 = 18e^{1.592}$

b. Circle the error in the "solution" and give a correct solution below.

$\frac{100}{18} = \frac{18e^{4k}}{18}$

$\frac{50}{9} = e^{4k}$

$\ln\left(\frac{50}{9}\right) = \ln e^{4k}$

$\ln\left(\frac{50}{9}\right) = 4k$

$\frac{\ln\left(\frac{50}{9}\right)}{4} = k$

$\boxed{k = .4287}$

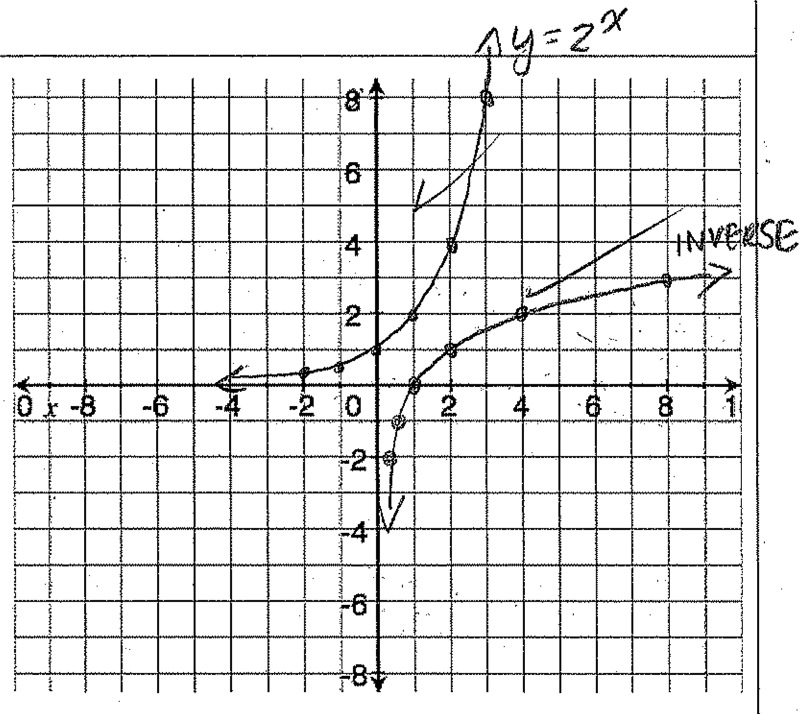
$100 = 18e^{4(.4287)}$

$100 = 18e^{1.7148}$

$100 = 100$ ✓

5. a. Make a table and accurately graph $y = 2^x$.

x	-2	-1	0	1	2	3	4
y	.25	.5	1	2	4	8	16



b. Make a table of values for the inverse of $y = 2^x$.

x	.25	.5	1	2	4	8	16
y	-2	-1	0	1	2	3	4

7

c. Sketch the graph of the inverse of $y = 2^x$. (Label which graph is which.)

d. Write the equation of the inverse of $y = 2^x$ in $y =$ form.

$\log_2 y = x$

$y = 2^x$ $x = 2^y$

$\log_2 x = y$

$\boxed{y = \log_2 x}$

So close!

6. Use the properties of logs to write the expression as a sum, difference, and/or multiple of logs. Simplify where possible.

$\log \frac{100\sqrt{y}}{x^2} =$

6

$\log 100\sqrt{y} - \log x^2 =$

$\boxed{2\sqrt{y} - \log x^2}$

$= \log 100 + \log\sqrt{y} - \log x^2$

$\boxed{2 + \tfrac{1}{2}\log y - 2\log x}$

7. Write the expression as the logarithm of a single quantity.

1.095

$\ln 3 + \frac{1}{3}\ln(4 - x^2) - \ln x$

$\boxed{\ln \frac{3(4-x^2)^{1/3}}{x}}$

8. Molly needs to find the range of $f(x) = \frac{e^x + 7}{2}$. She has a brilliant idea that she can use the inverse of $f(x)$ in a clever way to do this. Find $f^{-1}(x)$ and use it to find the range of $f(x)$.

5

$f^{-1}(x) = \frac{e^y + 7}{2}$

$y = \ln e^y + 7 - \ln 2$

$y = y + 7 - \ln 2$

$y = e^{\frac{x}{2}+7}$

$x = \frac{e^y + 7}{2}$

$2x = e^y + 7$

$2x - 7 = e^y$

$\ln 2x - 7 = y$

$y = \ln 2x - 7$

$2x - 7 = 0$

$2x = 7$

$\boxed{x \neq \frac{7}{2}}$ domain

$f^{-1}(x) = \ln(\cancel{2x-7})$

domain of inverse = range of original

domain $f(x)$: $x \neq \frac{7}{2}$

therefore, range $f(x)$: $x \neq \frac{7}{2}$

9. Select values for a and b, with $a > b$, such that $\log_b a$ is less than zero. Justify your choice for a and b.

5

$\log_b a = x$

$b^x = a$

✓ $\log_{-2} 2 = -1$

$\boxed{\begin{array}{l} a = 2 \\ b = -2 \end{array}}$ No negative bases...

✓ $-2^{-1} = 2$

$\frac{1}{2} = b$

$a = 4$

$\log_{\frac{1}{2}} 4 =$

$\frac{1}{2}^x = 4$

$\frac{1}{2^1}$

$2^1 = 4$

$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

$\frac{1}{2}^{-2} = 4$

$\boxed{-2}$

$\frac{4^{-2}}{1}$

$\frac{1}{1}$

$\frac{1}{4^2}$

$\frac{1}{4} = b$

$a = 16$

$\log_{\frac{1}{4}} 16 = \frac{1}{4}^x = 16$

$\frac{1}{4}^{-2} = 16$

this scribbling not originally on test! supercorrections

6

~~10.~~ Solve for x.

$$\log_a x + \log_a(x-2) = \log_a(x+4)$$

$\log_a x(x-2) = \log_a(x-4)$

$\log_a x^2 - 2x = \log_a(x-4)$

$x^2 - 2x = x - 4$

$+2x \quad +2x$

$x^2 = 3x - 4$

$x^2 - 3x + 4 = 0$

$\frac{3 \pm \sqrt{9-16}}{2} = \frac{3 \pm \sqrt{-7}}{2}$

$\cancel{\log_a}(x^2 - 2x) = \cancel{\log_a}(x+4)$

$x^2 - 2x = x + 4$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$\boxed{x = 4}$ ~~or -1~~

4

~~11.~~ Given that $\log_a b \approx 2.3219$, approximate

a. $\log_a \frac{1}{b^2}$

$= \log_a b^{-2}$

$a^{2.3219} = -2 \log_a b$

2.3219

$\times \; -2$

$\boxed{-4.6438}$

b. $\log_b a$

$\frac{\log_a a}{\log_a b} = \boxed{\frac{1}{2.3219}}$

c. $\log_a ab$

$\log_a a + \log_a b$

$= 1 + 2.3219 =$

$\boxed{3.3219}$

7

~~12.~~ Solve for x.

$$x^{\frac{3}{4}} = 8$$

$x^{\frac{1}{4} \cdot 3} = 8$

$x^{\frac{1}{4} \cdot 3} = 2^3$

$x = \sqrt[\frac{3}{4}]{2^3}$

$\boxed{x = \sqrt[\frac{3}{4}]{8}} = ?$

$\left(x^{\frac{3}{4}}\right)^{\frac{4}{3}} = (8)^{\frac{4}{3}}$

$x = 8^{\frac{4}{3}}$

$x = (2^3)^{\frac{4}{3}}$ $x = 4$ $\boxed{x = 16}$

7

~~13.~~ Suppose $g(x) = \log_7 x$ and $h(x) = 7^x$.

a. Find $g(h(x))$.

$\log_7 7^x =$

$x \cdot \log_7 7 = x \cdot 1 = \boxed{x}$

~~b.~~ Find $h(g(x))$.

$7^{\log_7 x} = \boxed{x}$

$7^{\log_7 x}$

c. What is the relationship between the functions g and h?

*inverse

Bonus: Choose values for a, b, and c so that the equation below is true. Justify your choice of a, b, and c.

$$\log_c\left[\log_a(\log_b c)\right] = 0$$